Number of Printed Pages:8

CLASS XII
MID TERM EXAMINATION 2023-24
MATHEMATICS (041)
SOLUTIONS SET A1/A2

Time:3hrs. **M.M.80**

General Instructions:

1.This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A (Multiple Choice Questions)
Each question carries 1 mark

1/18.The value of x, if the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular is
(a) -3 (b) 4 (c) 0 (d) 3
Ans.d

2/17.If A is a square matrix of order 3 and $A^2 = 3A$, then the value of $|A|$ is
(a) -3 (b) 3 (c) 9 (d) 27
Ans.d

3/16.If the matrix $\mathrm{A} = \begin{bmatrix} -2 & \mathrm{x-y} & 5 \\ 1 & 0 & 4 \\ \mathrm{x+y} & \mathrm{z} & 7 \end{bmatrix}$, is symmetric, the value of $\mathrm{x-y}$ is
(a)1 (b) -1 (c) 0 (d) 2
Ans.a

4/15.A is a 3×2 matrix whose elements are given by $a_{ij} = \frac{1}{2}|i-3j|$.The element a_{21} is
(a) 0 (b) $\frac{1}{2}$ (c) $\frac{5}{2}$ (d) $-\frac{1}{2}$
Ans.b

5/14.The number of equivalence relations on the set A ={ 2, 4, 6} containing (2,4) is/are
(a) 1 (b) 2 (c) 3 (d) 5
Ans.b

6/13. Which of the following functions from Z to itself is bijective?
(a) $f(x) = x^3$ (b) $f(x) = x+2$ (c) $f(x) = 2x+1$ (d) $f(x) = x^2 + x$
Ans.b

7/12.The value of $sin^{-1}\left(cos\frac{33\pi}{5}\right)$ is
(a) $\frac{3\pi}{5}$ (b) $\frac{-7\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $\frac{-\pi}{10}$
Ans.d

8/11.One branch of $\cos^{-1}$ other than the principal value branch corresponds to

(a) $\left[0, \frac{\pi}{2}\right]$ (b) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (c) (0, π) (d) [2π, 3π]

Ans.d

9/10. The function f(x) = $|cos x|$ is

(a)differentiable at x = (2n+1) $\frac{\pi}{2}$,nϵ Z

(b)continuous but not differentiable at x = (2n+1) $\frac{\pi}{2}$,nϵ Z

(c)differentiable for all x but not continuous at some x

(d)differentiable everywhere.

Ans.b

10/9. If $y^2 = ax^2 + b$, then $\frac{d^2y}{dx^2}$ =

(a) $\frac{ab}{x^3}$ (b) $\frac{x^3}{ab}$ (c) $\frac{ab}{y^2}$ (d) $\frac{ab}{y^3}$

Ans.d

11/8.The function $f(x) = \sin 3x$, $x \epsilon \left[0, \frac{\pi}{2}\right]$ is strictly decreasing in

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left(0, \frac{\pi}{6}\right)$ (c) $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$ (d) $\left(0, \frac{\pi}{4}\right)$

Ans.c

12/7.The maximum value of the function $f(x) = -(x-1)^2 + 10$ is

(a) 0 (b) 10 (c) 11 (d) 9

Ans.b

13/6.The value of $\int \frac{1}{x^2+2x+2} dx$ is

(a) $\sin^{-1}(x+1) + C$ (b) $\tan^{-1} x + C$ (c) $x \tan^{-1} x + C$ (d) $\tan^{-1}(x+1) + C$

Ans.d

14/5. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^5 + x^3 + x)\, dx$ is

(a) 0 (b) 1 (c) 3 (d) 4

Ans.a

15/4.The area(in square units) enclosed by curve $4x^2 + 9y^2 = 36$ is

(a) 4π (b) 6 π (c) 9 π (d) 36 π

Ans.b

16/3.The area (in square units) bounded by the lines $y = x + 1, x = 1, x = 3$ and x-axis is

(a) 6 (b) 8 (c) 7.5 (d) 2

Ans.a

17/2. Corner points of the feasible region determined by the system of linear constraints are $(0,10), (5,5), (15,15)$ and $(0,20)$. Let $\mathrm{Z} = \mathrm{px} + \mathrm{qy}$ where $\mathrm{p}, \mathrm{q} > 0$. Condition on p and q so that the maximum of Z occurs at both the points $(15,15)$ and $(0,20)$ is

(a) $\mathrm{p} = 2\mathrm{q}$ (b) $\mathrm{p} = \frac{q}{2}$ (c) $q = 3p$ (d) $\mathrm{p} = \mathrm{q}$

Ans. (c)

18/1. If the feasible region for a linear programming problem is bounded, then the objective function Z = ax + by has

(a) both a maximum and a minimum value on R. (b) only maximum value on R.

(c) only minimum value on R. (d)no maximum and minimum value on R

Ans.a

Question number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

19/20.Assertion (A) :Every differentiable function is continuous but converse is not true.

Reason (R) : Function f(x) = tan*x* is continuous on set of reals.

Ans.c

20/19. Assertion : $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}}\,dx = \frac{\pi}{2}$

Reason : $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

Ans.d

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21/25. Find $x,\ 0 < x < \frac{\pi}{2}$ such that $A + A' = I$ & $A = \begin{bmatrix} sinx & -cosx \\ cosx & sinx \end{bmatrix}$.

Ans. $\begin{bmatrix} sinx & -cosx \\ cosx & sinx \end{bmatrix} + \begin{bmatrix} sinx & cosx \\ -cosx & sinx \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow x = \frac{\pi}{6}$ 1+1

22/24.Find the value of $\tan^{-1} 1 + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$.

Or

Evaluate $\sec^2(\tan^{-1} 2) + \mathrm{cosec}^2(\cot^{-1} 3)$.

Ans. $\frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6} = \frac{3\pi}{4}$

Or

1+$(tan\tan^{-1} 2)^2 + 1 + (cot\cot^{-1} 3)^2$ =15 1+1

23/23. Find $\frac{dy}{dx}$ if sec (x + y) = xy.

Ans. sec (x + y)tan(x+y)(1+$\frac{dy}{dx}$)=y+x$\frac{dy}{dx}$ $\Longrightarrow \frac{dy}{dx} = \frac{y-\sec(x+y)\tan(x+y)}{\sec(x+y)\tan(x+y)-x}$ 1+1

24/22.If the value of $\int_0^a \frac{1}{4x^2+1}\,dx = \frac{\pi}{8}$, then find the value of a.

Or

Evaluate: $\int \frac{(x+1)(x+\log x)^2}{x}\,dx$

Ans.$\frac{1}{2}\tan^{-1} 2a = \frac{\pi}{8}, tan\frac{\pi}{4} = 2a, a = \frac{1}{2}$

Or

Put $x + \log x = t \Longrightarrow (1+\frac{1}{x})dx = dt\ I = \int t^2\,dt = \frac{t^3}{3} + C = \frac{(x+\log x)^3}{3} + C$ 1+1

25/21.For the curve y = 5x – 2x^3, if x increases at the rate of 2 units/sec, then how fast is the slope of curve changing when x = 3.

Ans. dy/dx=5-6x^2

$\frac{d}{dt}\left(\frac{dy}{dx}\right) = -12x.\frac{dx}{dt}$ $= -12 \,.\, (3) \,.\, (2) = -72$ units/sec. $\frac{1}{2} \times 4$

SECTION C

This section comprises of short answer (SA) type questions of 3 marks each.

26/31. Show that the function $f(x)$ defined as $f(x) = \begin{cases} 3x-2; & 0 < x \le 1 \\ 2x^2 - x; & 1 < x \le 2 \\ 5x - 4; & x > 2 \end{cases}$ is not differentiable at $x = 2$.

Ans. $Lf'(2) = \lim_{h\to 0} \frac{f(2-h) - f(2)}{-h} = \lim_{h\to 0} \frac{2(2-h)^2 - (2-h) - 6}{-h} = 7$ & $Rf'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{5(2+h)-4-6}{h} = 5$. Therefore, $Lf'(2) \neq Rf'(2)$.not differentiable at x=2 1.5+1.5

27/30. If $x = \tan\left(\frac{1}{a}\log y\right)$, then show that $(1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$.

Or

If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, find the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{3}$

Ans. $\log y = a \tan^{-1} x \implies y = e^{a\tan^{-1}x} \Rightarrow \frac{dy}{dx} = \frac{ay}{1+x^2} \Rightarrow (1+x^2)\frac{dy}{dx} = ay \Rightarrow$ $(1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$.

Or

$\frac{dx}{dt}$ =a(tcost), $\frac{dy}{dt}$ =a(t sint), $\frac{dy}{dx}$=tant ,$\left.\frac{d^2y}{dx^2}\right) = \left.\frac{sec^3t}{at}\right)_{\frac{\pi}{3}} = \frac{24}{a\pi}$, 1+1+1

28/29.Show that $y = \log(1+x) - \frac{2x}{2+x}, x > -1$ is an increasing function of x throughout its domain.

Or

Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.

Ans. $y = f(x) = \log(1+x) - \frac{2x}{2+x} \Rightarrow f'(x) = \frac{1}{1+x} - 2\left\{\frac{(2+x).1-x}{(2+x)^2}\right\} = \frac{x^2}{((1+x)((2+x)^2))} \ge 0$

$\Rightarrow y$ is increasing function.

Or

$f'(x) = 12x^2 - 36x + 27 = 3(4x^2 - 12x + 9) = 3(2x-3)^2$ $f'(x) = 0 \Rightarrow x = \frac{3}{2}$ (critical point)

Since f′ (x) > 0 for all x> $\frac{3}{2}$ & all x< $\frac{3}{2}$ so it is a point of inflexion i.e., neither a point of maxima nor a point of minima & being only critical point f(x) has neither maxima nor minima.

1+1+1

29.Find a matrix X such that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Ans. X=$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $a + 4b = -7, c + 4d = 2, 2a + 5b - 8, 2c + 5d = 4, X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$ 1+1+1

/28. Find a matrix A such that $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$.

Ans.A=$[a \quad b \quad c]$, $4a=-4$, $4b=8,4c=4$, $[-1 \quad 2 \quad 1]$ 1+1+1

30/27.Show that the function $f: R-\{3\} \rightarrow R-\{2\}$ defined by $f(x)=\frac{2x+3}{x-3}$, is one-one and onto.

Or

Determine whether the relation **R1** defined on the set R of all real numbers as
$\boldsymbol{R1}=\left\{(a,b): a,b \in R \text{ and } a-b+\sqrt{3} \in S, \text{ where } S \text{ is the set of all irrational numbers } \right\}$
is reflexive and symmetric.

Ans.

Let $x_1, x_2 \in A=R-\{3\}$
Let $f(x_1)=f(x_2)$

$\Rightarrow \frac{2x_1+3}{x_1-3}=\frac{2x_2+3}{x_2-3}$
$\Rightarrow (2x_1+3)(x_2-3)=(2x_2+3)(x_1-3)$
$\Rightarrow (2x_1x_2-6x_1+3x_2-9)=(2x_1x_2-6x_2+3x_1-9)$
$\Rightarrow -6x_1+3x_2=-6x_2+3x_1$
$\Rightarrow 9x_1=9x_2$
$\Rightarrow x_1=x_2$

Now $f(x_1)=f(x_2) \Rightarrow x_1=x_2$ so one-one

$y=\frac{2x+3}{x-3}$
$\Rightarrow xy-3y=2x+3$
$\Rightarrow xy-2x=3y+3$
$\Rightarrow x(y-2)=3(y+1)$
$\Rightarrow x=\frac{3(y+1)}{(y-2)}$ .... which is defined for all real values of y except 2 i.e ∈R−{2} which is same as given set = R−{2}Codomain=Range so onto

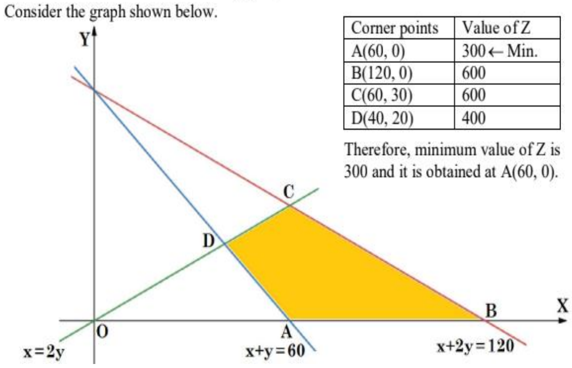
Or

(a,a)∈ $R1$ as $a-a+\sqrt{3} \in \boldsymbol{S}$ for all a∈ $\boldsymbol{R}$, so R1 is reflexive.Any valid example for R1 not symmetric. $1\frac{1}{2}+1\frac{1}{2}$

31/26.Solve the following Linear Programming Problem graphically:
Minimize : Z =5x +10y Subject to constraints : x +2y≤ 120, x+ y≥ 60, x −2y≥ 0,x≥ 0, y≥ 0.

Ans.

Consider the graph shown below.



Corner points	Value of Z
A(60, 0)	300← Min.
B(120, 0)	600
C(60, 30)	600
D(40, 20)	400

Therefore, minimum value of Z is 300 and it is obtained at A(60, 0).

1+1+1

SECTION D

This section comprises of long answer-type questions (LA) of 5 marks each.

32/35. Solve the system of equations using matrix method
$x+2y-3z=-4,\ 2x+3y+2z=2, 3x-3y-4z=11$.

Or

Use the product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of linear equations

$x - y + 2z = 1,\ 2y - 3z = 1, 3x - 2y + 4z = 2$.

Ans. AX=B , $\begin{bmatrix}1 & 2 & -3\\ 2 & 3 & 2\\ 3 & -3 & -4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}-4\\ 2\\ 11\end{bmatrix}$,

$X = A^{-1}B = \frac{1}{67}\begin{bmatrix}-6 & 17 & 13\\ 14 & 5 & -8\\ -15 & 9 & -1\end{bmatrix}\begin{bmatrix}-4\\ 2\\ 11\end{bmatrix} = \frac{1}{67}\begin{bmatrix}201\\ -134\\ 67\end{bmatrix}$ $x = 3\,, y = -2\,, z = 1$ 1+2+2

Or

AB = $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ $AB = I, so\ \ A^{-1} = B$

$X = A^{-1}C = BC = \begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}\begin{bmatrix}1\\ 1\\ 2\end{bmatrix} = \begin{bmatrix}0\\ 5\\ 3\end{bmatrix} \Rightarrow x = 0, y = 5, z = 3$

33. If $y^x + x^y + x^x = a^b$, then find $\frac{dy}{dx}$.

Ans. $u = x^y, v = y^x$ and $w = x^x$ so that $u + v + w = a^b \Rightarrow \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0$

$u = x^y \Rightarrow logu = y\,logx \Rightarrow \frac{du}{dx} = x^y\left(\frac{y}{x} + logx\,\frac{dy}{dx}\right), \frac{dv}{dx} = y^x\left(\frac{x}{y}\frac{dy}{dx} + logy\right)$ & $\frac{dw}{dx} =$

$x^x(1 + logx) \Rightarrow \frac{dy}{dx} = \frac{-[y^x\log y + yx^{y-1} + x^x(1+\log x)]}{xy^{x-1} + x^y\log x}$. 1+1+1+2

/34. Differentiate $\left(x^{xcosx} + \frac{x^2+1}{x^2-1}\right) w.r.t.x.$

Ans. $y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}, \log u = xcosx\log x \Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{xcosx}{x} + \log x\,.(cosx - x\sin x)$

$\Rightarrow \frac{du}{dx} = x^{xcosx}(cosx + \log x\,(cosx - xsinx))$

& $\frac{dv}{dx} = \frac{-4x}{(x^2-1)^2}, \frac{dy}{dx} = x^{xcosx}(cosx + \log x\,(cosx - xsinx)) - \frac{4x}{(x^2-1)^2}$. 1+2+2

34/33. Using properties of definite integrals, evaluate $\int_0^{\pi}\frac{x\tan x\,dx}{secx+tanx}$.

Or

Evaluate $\int\frac{x^4dx}{(x-1)(x^2+1)}$.

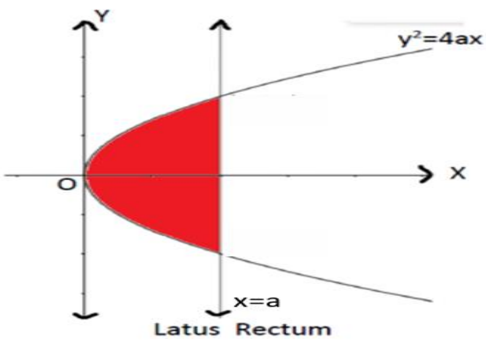
Ans.I = $\int_0^{\pi}\frac{(\pi-x)\tan x\,dx}{sec(\pi-x)+tan(\pi-x)}$, $2I = \pi\int_0^{\pi}\frac{sinxdx}{1+sinx} = \pi\int_0^{\pi}(tanxsecx - sec^2x + 1)dx\,, I =$ $\pi\left(\frac{\pi}{2} - 1\right)$

Or

I=$\int(x + 1 + \frac{1}{2(x-1)} - \frac{x}{2(x^2+1)} - \frac{1}{2(x^2+1)})dx = \frac{x^2}{2} + x + \frac{1}{2}log|x - 1| - \frac{1}{4}log|x^2 + 1| -$ $\frac{1}{2}\tan^{-1}x + c$ 1+1+2+1

35/32. Using integration, find the area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum.

Ans.



Required area $= 2\int_0^a 2\sqrt{ax}\ \ dx$

$= 4\sqrt{a}\int_0^a \sqrt{x}\ \ dx$

$= 4\sqrt{a}\left|\frac{2x^{3/2}}{3}\right|_0^a$

$= \frac{8}{3}a\sqrt{a}\ \sqrt{a}$

$= \frac{8}{3}a^2$ sq units 2+1+2

SECTION E

(This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. Questions 36 & 38 have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. Question 37 has two sub parts of 2 marks each.)

36/38.Kriti and Kirat are two friends studying in class XII in a school at Chandigarh. While doing their Mathematics project on Relations and Functions they have to collect the names of three metro cities and three cities other than metro cities and present the name of cities in the form of sets. They have collected the name of cities and write in the form of sets given as follows:

A={three metro cities of India}= { Delhi, Mumbai, Bangalore} and

B = {three non metro cities of India} = { Patiala, Agra, Jaipur,}

Answer the following questions using the above information.

(i)How many functions are possible from A to B.

(ii)How many relations are possible from A to B.

(iii)Find the number of reflexive relations on set A.

Or

Let a function f: $A \to B$ be defined as f = {(Delhi, Agra), (Mumbai, Jaipur),(Bangalore,Patiala)}. Check if f is one-one & onto.Justify.

Ans.(i) Number of functions= $3^3 = 27$

(iii) Number of relations= $2^9 = 512$

(iii)Number of reflexive relations $= 2^{3^2-3} = \ 2^6 = 64$

Or

f is one-one as each element in A has a unique image in B .f(A)=B so onto . 1+1+2

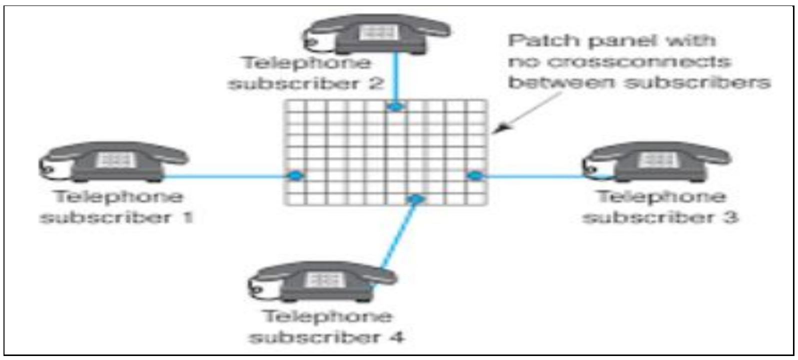
37/37.A school wants to award its students for the values of honesty, regularity and hard work with a total cash award of Rs 6000. Three times the award money for hard work added to that

given for honesty amounts to Rs 11000. The award money given for honesty and hard work together is double the one given for regularity.
(i) If Rs x is awarded to honesty, Rs y to regularity and Rs z awarded to hard work, then what is the matrix equation AX=B representing the above situation ? Find determinant of transpose of A.
(ii)Find A(Adj A).

Ans.(i) $\begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{3} \\ \mathbf{1} & \mathbf{-2} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \end{bmatrix} = \begin{bmatrix} \mathbf{6000} \\ \mathbf{11000} \\ \mathbf{0} \end{bmatrix}$ $|A^/|$ =6

(ii)6I 2+2

38/36.A telephone company in a town has 500 subscribers on its list and collects fixed charges of Rs.300 per subscriber per year. The company proposes to increase the annual subscription and it is believed that for every increase of Re.1 one subscriber will discontinue the service.



(i)If Rs x be the annual subscription then find the total revenue R(x) of the company after increment.
(ii)Find the derivative of R(x).
(iii)How much fee the company should increase to have maximum profit?

or

(iii) Find the maximum profit that the company can make if the profit function is given by P(x) = 41 + 24x – 18x^2.
Ans.(i) R(x)= –x^2 + 800x
(ii)R$^/$ (x)= –2x+800
(iii) **R$^/$ (x)=0** gives x=400 , **R$^{//}$(x)=–2<0** so max at Rs 100 increase.
Or
(iii)P$^/$(x)=24-36x, **P$^/$ (x)=0** gives x=2/3 , **P$^{//}$(x)=-36<0** so max at profit P(2/3)=Rs 49